PAPER CODE NO. **PHYS393**	EXAMINER: Dr. K. M. Hock DEPARTMENT: Physics	TEL. NO. 01925-603853

JANUARY EXAMINATIONS 2012

Bachelor of Science: Year 3
Master of Physics: Year 3
Master of Physics: Year 4

STATISTICAL AND LOW TEMPERATURE PHYSICS

TIME ALLOWED: 3 hours

INSTRUCTIONS TO CANDIDATES

Answer all questions.

Question 1 carries 50% of the total marks.

Questions 2 and 3 each carry 25% of the total marks.

Answer either part (a) or part (b) of questions 2 and 3.

In the event of a student answering both parts of an either/or question and not clearly crossing out one answer, only the answer to part (a) of the question will be marked.

The marks allotted to each part of a question are indicated in square brackets.

All symbols have their usual meanings unless otherwise stated.

Question 1.

(a) One mole of a spin ½ salt sits in a 1 T magnetic field, at a temperature of 1 K.

i) Find the energies of the magnetic levels. **[2]**

ii) Write down the formulae for the most probable macrostate. **[2]**

iii) Calculate the Boltzmann factor for each level. **[2]**

iv) What is the ratio of the populations in the two levels? **[2]**

v) Find the populations (i.e. the number of moles in each level). **[2]**

(b) One mole of Neon (relative atomic mass = 20) gas at 300 K is in a cube of side L.

i) Find the average energy of the atoms. **[2]**

ii) Find the corresponding wavevector. **[3]**

iii) If L is 30 cm, find the spacing between wavevectors. **[2]**

iv) Why do we need density of states? **[3]**

(c) A spin ½ salt sits in a magnetic field. At high temperatures, there are 0.1 mole of spin ½ ions at each of the two magnetic energy levels.

i) At 1 K, the number of the ions at the lower level increased by 50%. Find the population at each level. **[3]**

ii) Find the energy difference between the levels. **[3]**

iii) How much heat is given out? **[4]**

(d)

i) The presence of a magnetic field in a macroscopic wavefunction of electrons must produce a current. How does this explain the Meissner's effect? **[2]**

ii) Discuss how this gives rise to London's penetration depth. **[4]**

iii) Sketch the heat capacity versus temperature graph for a superconductor, above and below the transition temperature. How does this suggest the existence of an energy gap? **[4]**

(e) A body can move with zero resistance through superfluid ^4He.

i) What excitations are possible in superfluid ^4He? **[2]**

ii) Consider an excitation of energy E and momentum p. The body's velocity must be above E/p before excitation is possible. What are the conservation laws that lead to this? **[4]**

iii) Sketch the dispersion relation of the excitations in superfluid ^4He. Draw the line with gradient equal to the minimum E/p. **[2]**

iv) Why does the body experience no resistance when its velocity is below the minimum E/p? **[2]**

Question 2. Answer **<u>either</u>** (a) **<u>or</u>** (b)

2(a)

One mole of copper is at 1 K. Each atom supplies two conduction electrons.

i) Assuming that the electrons behave like an ideal gas, write down the expression for the average kinetic energy of the electrons. Hence find the heat capacity. **[3]**

ii) At 1 K, the measured heat capacity is 0.6 mJ/K. Explain why it is different from the answer in (i). **[2]**

iii) With the help of a graph, estimate the energy range of the electrons that are excited above the Fermi energy at temperature T. **[4]**

iv) Using the density of states for the ideal gas,

$$g(\varepsilon) = \frac{4m\pi V}{h^3}\sqrt{2m\varepsilon},$$

derive an expression for the number, n, of excited electrons. (Molar volume of copper is 7.11 cm^3).

[4]

v) Why is it reasonable to assume that the excited electrons in (iv) behave like the ideal gas? **[4]**

vi) Derive an expression for the heat capacity of the excited electrons using the ideal gas formula from (i). **[4]**

vii) Find the Fermi energy. Using the expression from (vi), calculate the heat capacity for copper at 1 K. Compare with values in (i) and (ii). What does this suggest? **[4]**

2 (b)

Dilution cooling makes use of a mixture of liquid ^{3}He in liquid ^{4}He.

i) Sketch a phase diagram of the mixture. Label the axes, the lambda line and the phase separation region. **[5]**

ii) Describe qualitatively what happens when the temperature of a 50% mixture falls just below the phase separation curve. **[5]**

iii) Describe qualitatively what happens when the temperature of this mixture reaches a few milliKelvin. **[3]**

iv) Sketch a diagram to explain qualitatively how the ^{3}He could be removed from the bottom layer. **[6]**

v) Why is this necessary? What would then happen to the top layer, and what must we do about it? **[6]**

Question 3.

Answer **either** (a) **or** (b)

(a)

i) Why is the Bose-Einstein condensate a good candidate for explaining superfluidity and superconductivity? **[5]**

ii) Using a sketch of the Fermi-Dirac distribution graph, explain what happens to the chemical potential μ of a boson gas as temperature falls to 0 K? **[5]**

iii) In terms of the density of states g(ε), where ε is the energy, write down the expression for the number of particles N. Explain when and why the number of excited bosons may be written as

$$N_{ex} = \int_0^\infty \frac{g(\varepsilon)d\varepsilon}{\exp(\varepsilon / k_B T) - 1},$$

where T is the temperature. **[5]**

iv) Given that the solution is

$$N_{ex} = \left(\frac{2\pi m k_B T}{h^2} \right)^{3/2} 2.612V,$$

where V is the volume that contains the N particles, explain how to find the condensation temperature T_{BE}. Determine the value of T_{BE} for liquid ^{4}He, with molar volume 27.58 cm^3. **[5]**

v) In terms of g(ε), write down the expression for the energy U below T_{BE}. Given that

$$U = 0.7704 k_B N \frac{T^{5/2}}{T_{BE}^{\ 3/2}}$$

is the solution, derive the heat capacity C. Calculate the value of C for one mole of ^{4}He at T_{BE}, and sketch the graph of C versus T. **[5]**

3 (b)

Superconductivity in a metal is explained by the forming of Cooper pairs. A Cooper pair is a pair of electrons that for some reason are able to attract each other.

i) Describe one experimental result that is consistent with such a binding energy. **[5]**

ii) Explain qualitatively one experiment which suggests that superconductivity has something to do with lattice vibration. **[5]**

iii) Sketch a picture to explain how movements of the atoms could produce an attraction between two electrons. **[5]**

iv) Even if this attraction is possible, it would be very weak. Using the Fermi level, explain why the electrons do not just come apart. **[5]**

v) How is the forming of Cooper pairs consistent with the macroscopic wavefunction that is used to explain the Meissner's effect? **[5]**

CONSTANTS

Speed of light in vacuum	c	=	3.00×10^{8} ms^{-1}
Permeability of vacuum	μ_0	=	$4\pi \times 10^{-7}$ Hm^{-1}
		=	$4\pi \times 10^{-7}$ VsA^{-1}m^{-1}
Permittivity of vacuum	ε_0	=	8.85×10^{-12} Fm^{-1}
		=	8.85×10^{-12} AsV^{-1}m^{-1}
Elementary charge	e	=	1.60×10^{-19} C
Planck constant	h	=	6.63×10^{-34} Js
	$h/2\pi = \hbar$	=	1.05×10^{-34} Js
Avogadro constant	N_A	=	6.02×10^{23} mol^{-1}
Boltzmann constant	k_B	=	1.38×10^{-23} JK^{-1}
Gas constant	R	=	8.31 JK^{-1}mol^{-1}
Unified atomic mass constant	m_u	=	1.66×10^{-27} kg
		=	931.5 MeVc^{-2}
Electron mass	m_e	=	9.11×10^{-31} kg
Proton mass	m_p	=	1.67×10^{-27} kg
Gravitational constant	G	=	6.67×10^{-11} Nm^{2}kg^{-2}
Acceleration due to gravity	g	=	9.8 ms^{-2}
Bohr magneton	μ_B	=	9.27×10^{-24} JT^{-1}